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PROBLEMS.

44 **Proposed by I. J. SCHWATT, Ph. D., Professor of Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania.**

(1). If from the middle point M of the side BC of the triangle ABC a parallel to the bisector AF of the external angle to ABC is drawn to meet AB at K, the point K divides then the side AB in KA

$$=\tfrac{1}{2}(AB+AC) \text{ and } KB=\tfrac{1}{2}(AB-AC).$$

(2). If K is joined to the extremity D of the diameter perpendicular to BC then is KD perpendicular to AB.

45. **Proposed by B. F. BURLESON, Oneida Castle, New York.**

Determine the radius of a circle circumscribing three tangent circles of radii $a=15$, $b=17$, and $c=19$.

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CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

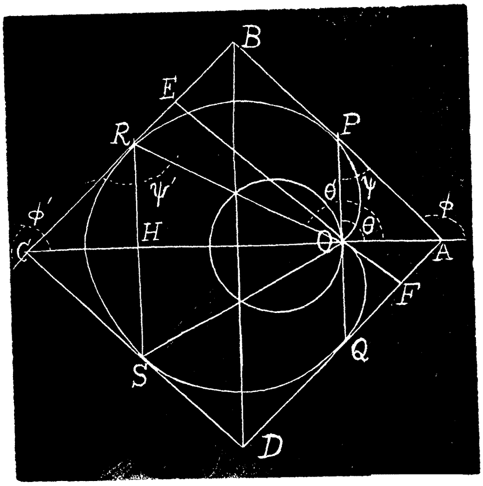
SOLUTIONS OF PROBLEMS.

30. **Proposed by E. W. NICHOLS, Professor of Mathematics in the Virginia Military Institute, Lexington, Virginia.**

Given the cardioid $r=a$ $(1-\cos\theta)$; find the area of its circumscribing square formed by tangents making angles of 45° with its axis.

I. Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $OPRSQ$ be the cardioid. Draw PQ through the cusp perpendicular to the initial line AC. From the property of the cardioid the angle APO, made by the tangent and radius vector at P, $=\frac{1}{2}\angle POA$. But $\angle AOP=\frac{1}{2}\pi$. $\therefore$ $\angle OPA=\angle OAP=\frac{1}{4}\pi$. $\therefore$ the tangents BA, DA at the points P, Q are inclined at an angle of 45° to the axis and are perpendicular to each other. Draw the radii vectors OR, OS, making the $\angle ROP=\angle SOQ=\frac{1}{3}\pi$, and draw the tangents CB, CD at the points R, S. Then $\angle ROP=\angle SOQ=\frac{1}{3}\pi$, $\angle OPB=\angle OQD=\frac{3}{4}\pi$, $\angle ORB=\angle OSD=\frac{5}{12}\pi$.



$$\therefore\ \angle ROP+\angle OPB+\angle ORB=\angle SOQ+\angle OQD+\angle OSD=\tfrac{3}{2}\pi.$$

$\therefore$ $\angle B=\angle D=\frac{1}{2}\pi$, an $ABCD$ is the required square.

Now $OR=a(1-\cos\frac{5}{6}\pi)=a(1+\frac{1}{2}\sqrt{3})$, $OQ=a(1-\cos\frac{3}{2}\pi)=a$.

$$OE=OR\sin ORB=OR\sin 75^\circ=\frac{a(2+\sqrt{3})}{2}+\frac{1+\sqrt{3}}{2\sqrt{2}}=\frac{5+3\sqrt{3}}{4\sqrt{2}}a.$$

$$OF=OQ\sin OQF=OQ\sin 45^\circ=\frac{a}{\sqrt{2}}.$$

$$AB=OE+OF=\frac{3(3+\sqrt{3})}{4\sqrt{2}}a.$$

Area square$=AB^2=\dfrac{27(2+\sqrt{3})}{16}a^2$. A solution without the use of Calculus.

II. Solution by Cadet A. R. GATEWOOD, Virginia Military Institute, Lexington, Virginia; and COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tennessee.

$$\text{Tan}\,\Psi=r\Big/\frac{dr}{d\theta}\cdot\frac{dr}{d\theta}=a\sin\theta.\quad\therefore\ \tan\Psi=\frac{r}{a\sin\theta}=\frac{2a\sin^2\frac{\theta}{2}}{2a\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$=\tan\frac{\theta}{2}.\quad\therefore\ \Psi=\frac{\theta}{2}.\quad \phi=\Psi+\theta=\frac{\theta}{2}+\theta=\frac{3\theta}{2}.$$

Now when tangents make angles of 45° each with the initial line, $\phi=135^\circ$, $\phi'=225^\circ$. $\therefore$ $\theta=90^\circ$, $\theta'=150^\circ$, $\angle ROH=30^\circ$.

When $\theta=90^\circ$, $OA=OP=a$. When $\theta=150^\circ$, $RO=a(1-\cos150^\circ)=a\left(1+\frac{\sqrt{3}}{2}\right)$,

$$CH=RH=RO\sin30^\circ=\tfrac{1}{2}a\left(1+\frac{\sqrt{3}}{2}\right),\ HO=RO\cos30^\circ=\frac{\sqrt{3}}{2}a\left(1+\frac{\sqrt{3}}{2}\right).$$

Now $CA=CH+HO+OA=\frac{1}{2}a\left(1+\frac{\sqrt{3}}{2}\right)+\frac{\sqrt{3}}{2}a\left(1+\frac{\sqrt{3}}{2}\right)+a=\frac{3}{4}a(3+\sqrt{3})$.

$ABCD=BA^2=\frac{1}{2}CA^2=\frac{1}{2}\left[\frac{3}{4}a\,(3+\sqrt{3})\right]^2=\frac{27}{16}(2+\sqrt{3})a^2$ which is the area of the circumscribed square.

III. Solution by F. P. MATZ, Ph. D., New Windsor College, New Windsor, Maryland; ALFRED HUME, C. E., University of Mississippi, P. O., Mississippi; and J. SCHEFFER, A. M., Hagerstown, Maryland.

Since the sides of the circumscribing square are to intersect the axis at angles of 45°, we have from *Todhunter's Differential Calculus*, p. 304, Art. 278, that

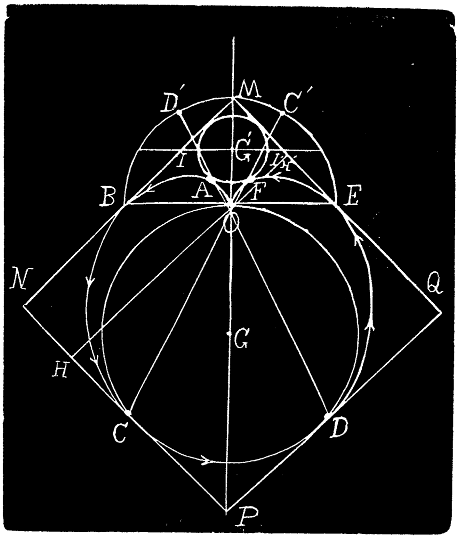
$$\tan\Psi=\frac{dy}{dx}=\frac{\sin\theta(dr/d\theta)+r\cos\theta}{\cos\theta(dr/d\theta)-r\sin\theta},$$

$$=\frac{\sin\theta(a\sin\theta)+a(1-\cos\theta)\cos\theta}{\cos\theta(a\sin\theta)-a(1-\cos\theta)\sin\theta}=\pm1\ldots(1).$$

From (1) we have, respectively:

$\tan\frac{3}{2}\theta=+1$. $\therefore$ $\theta=\frac{1}{6}\pi$, $\frac{5}{6}\pi$, $\frac{9}{6}\pi$, etc., and $\tan\frac{3}{2}\theta=-1$. $\therefore$ $\theta=\frac{3}{6}\pi$, $\frac{7}{6}\pi$, $\frac{11}{6}\pi$, etc.

Taken in the order of their magnitude, these values of θ represent the angular position of the points A, B, C, D, E, and F, with respect to the origin of polar co-ordinates and the axis of the cardioid If any tangents to the cardioid be drawn through *these* points, such tangents make angles of 45° with the axis MP. When the origin of polar co-ordinates is at O, the radius vectors of the points already specified becomes respectively:



$$OA=\tfrac{1}{2}(2-\sqrt{3})a,\ OB=a,$$
$$OC'=\tfrac{1}{2}(2+\sqrt{3})a,\ OD=\tfrac{1}{2}(2+\sqrt{3})a,$$
$$OE=a, \text{ and } OF=\tfrac{1}{2}(2-\sqrt{3})a.$$

Consequently the area of the required circumscribing square

$$=(MN)^2=(OH+OH')^2$$
$$=\tfrac{9}{32}(3+\sqrt{3})^2a,=\tfrac{27}{16}(2+\sqrt{3})a^2\ \ldots.(2);$$

and of this square, the diagonal $MP=\frac{3}{4}(3+\sqrt{3})a$. Since the *diagonal* $II'=\frac{3}{4}(3+\sqrt{3})a$, the area of the square the center of the inscribed circle of which is at $G'=(MI)^2=\frac{9}{32}(3-\sqrt{3})^2a^2$, $=\frac{27}{16}(2-\sqrt{3})a^2\ \ldots.(2)$.

Represent the area of the *larger* square by A and that of the *smaller* square by A'; then from the results given, $A:A'::(2+\sqrt{3}):(2-\sqrt{3})::OC:OF$.

IV. Second Solution by Professors F. P. MATZ; and C. E. WHITE, Trafalgar, Indiana.

The *pedal equation* of the cardioid in consideration, is $p^2=r^3/2a$; that is, for the points B, C, D, and E, we have respectively: $p_B=a/\sqrt{2}$, $p_C=\frac{1}{4}a\sqrt{(26+15\sqrt{3})}$, $p_D=\frac{1}{4}a\sqrt{(26+15\sqrt{3})}$, and $p_E=a/\sqrt{2}$. In order that the quadrilateral circumscribing the cardioid may be a square, we must have $p_B+p_D=p_C+p_E$; and this condition is fulfilled. Hence the *required* area becomes $A=(p_B+p_D)^2=(p_C+p_E)^2=\frac{27}{16}(2+\sqrt{3})a^2\ \ldots.(1)$,

while the area of the *smaller* square becomes

$$A'=(p_B-p_F)^2=(p_E-p_A)^2=\tfrac{27}{16}(2-\sqrt{3})a^2\ \ldots.(2).$$

NOTE—By using the equation $p=2a\sin^3\phi$, in which $\phi=\frac{1}{2}\theta$, a *third* solution can be made. Sufficient data are given in the problem to enable us to make a *fourth* solution, without having recourse to the differential calculus.

Also solved by *O. W. Anthony*, *H. W. Draughon*. and *J. B. Faught*. We regret that *Professor Faught's* solution was mislaid and could not be considered in selecting papers for publication.

PROBLEMS.

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41. Proposed by F. P. MATZ, M. Sc., Ph.D.. Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

The *closed portion* of the curve known as "The Cocked Hat," equation